XII. *On the Tides of the Arctic Seas.*

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Part V. *On the Tides of Refuge Cove, Wellington Channel.*

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THE following observations, like those at Northumberland Sound, were made on board H.M.S. 'Assistance,' under the command of Sir EDWARD BELCHER, R.N., K.C.B. They were made from 16th September to 11th October 1853. Although the period of observation is so short, yet, owing to the fact that it was the time of Equinox, some useful information has been obtained as to the Lunar Diurnal Tide at this Station.

The position of Refuge Cove is

Lat. 75° 31′ N.

Long. 92° 10′ W.

The following Table contains the Height of each High and Low Water, and the Height of the Diurnal Tide, calculated by the second difference of the heights.

TABLE I.—Refuge Cove.

Time.	High Water. Height.	Low Water. Height.	Diurnal Tide at High Water.	Diurnal Tide at Low Water.
1853. h m	ft. in.	ft. in.	ft.	ft.
Sept. 17. 6 30 A.M.	......	6 0		
17. 12 30 P.M.	10 6½			
17. 6 25 „	......	5 3	......	0·313
18. 1 0 A.M.	11 8½	......	0·573	
18. 7 30 „	......	5 9	......	0·271
18. 12 50 P.M.	10 7	......	0·477	
18. 7 0 „	......	5 2	......	0·224
19. 1 30 A.M.	11 4	......	0·393	
19. 7 40 „	......	5 6	......	0·193
19. 1 45 P.M.	10 6	......	0·271	
19. 7 55 „	......	5 2	......	0·167
20. 2 0 A.M.	10 11	......	0·135	
20. 8 20 „	......	5 8	......	0·094
20. 2 10 P.M.	10 11	......	0·057	
20. 8 0 „	......	5 10	......	0·005
21. 2 5 A.M.	11 0	......	0·055	
21. 8 30 „	......	6 0	......	0·083
21. 2 25 P.M.	10 9	......	0·026	
21. 8 40 „	......	6 5	......	0·203
22. 2 54 A.M.	10 7½	......	0·046	
22. 8 45 „	......	6 0	......	0·323
22. 3 30 P.M.	10 8	......	0·120	
22. 9 45 „	......	6 11	......	0·234

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TABLE I. (continued).

Time.			High Water. Height.	Low Water. Height.	Diurnal Tide at High Water.	Diurnal Tide at Low Water.
1853.		h m	ft. in.	ft. in.	ft.	ft.
Sept.	23.	3 30 A.M.	10 2	......	0·112	
	23.	9 50 „	......	6 2	......	0·439
	23.	3 55 P.M.	10 4	......	0·005	
	23.	10 30 „	......	7 6	......	0·516
	24.	4 0 A.M.	10 7	......	0·041	
	24.	10 30 „	......	6 9	......	0·594
	24.	5 0 P.M.	10 3	......	0·125	
	24.	11 10 „	......	8 0	......	0·604
	25.	5 10 A.M.	9 4	......	0·266	
	25.	10 20 „	......	6 10	......	0·687
	25.	5 30 P.M.	9 10	......	0·234	
	26.	12 30 A.M.	......	8 6	......	0·698
	26.	5 45 „	9 6	......	0·240	
	26.	12 30 P.M.	......	7 3	......	0·661
	26.	6 30 „	9 10	......	0·401	
	27.	1 15 A.M.	......	8 6	......	0·651
	27.	7 0 „	8 6	......	0·531	
	27.	1 30 P.M.	......	7 1	......	0·635
	27.	7 30 „	9 7	......	0·594	
	28.	3 0 A.M.	......	8 3	......	0·578
	28.	8 0 „	8 8	......	0·740	
	28.	3 30 P.M.	......	7 1	......	0·516
	28.	10 5 „	10 10	......	0·849	
	29.	4 0 A.M.	......	7 11	......	0·455
	29.	10 25 „	9 4	......	0·799	
	29.	4 0 P.M.	......	6 10	......	0·375
	29.	10 40 „	11 0	......	0·740	
	30.	5 0 A.M.	......	7 2½	......	0·328
	30.	10 45 „	9 10½	......	0·734	
	30.	4 45 P.M.	......	6 5	......	0·344
	30.	11 45 „	11 10	......	0·672	
Oct.	1.	6 0 A.M.	......	7 1	......	0·331
	1.	11 0 „	11 0	......	0·565	
	1.	5 15 P.M.	......	6 4	......	0·208
	1.	11 45 „	12 3	......	0·490	
	2.	5 0 A.M.	......	6 7	......	0·182
	2.	12 30 P.M.	11 5	......	0·354	
	2.	6 15 „	......	6 3	......	0·115
	3.	12 20 A.M.	12 2	......	0·224	
	3.	6 0 „	......	6 8	......	0·110
	3.	1 0 P.M.	12 6	......	0·078	
	3.	6 45 „	......	7 4	......	0·099
	3.	Midnight	13 0	......	0·088	
	4.	7 15 A.M.	......	7 4	......	0·188
	4.	Noon	13 0	......	0·094	
	4.	8 0 P.M.	......	7 10	......	0·297
	5.	1 0 A.M.	13 3	......	0·094	
	5.	8 30 „	......	7 0	......	0·380
	5.	1 28 P.M.	13 0	......	0·094	
	5.	8 20 „	......	7 9	......	0·386
	6.	1 40 A.M.	13 0	......	0·135	
	6.	8 30 „	......	6 11	......	0·380
	6.	1 0 P.M.	12 6	......	0·182	
	6.	9 30 „	......	7 8	......	0·386
	7.	4 0 A.M.	12 11	......	0·125	
	7.	9 0 „	......	7 1	......	0·484
	7.	3 0 P.M.	12 9	......	0·026	
	7.	9 45 „	......	8 5	......	0·615

TABLE I. (continued).

Time.				High Water. Height.	Low Water. Height.	Diurnal Tide at High Water.	Diurnal Tide at Low Water.
1853.		h m		ft. in.	ft. in.	ft.	ft.
Oct.	8.	4 0	A.M.	12 6	......	0·031	
	8.	10 0	,,	......	7 2	......	0·688
	8.	5 0	P.M.	12 3	......	0·172	
	8.	10 50	,,	......	8 8	......	0·755
	9.	4 15	A.M.	11 5	......	0·370	
	9.	10 50	,,	......	7 3	......	0·823
	9.	4 40	P.M.	12 2	......	0·547	
	10.	12 10	A.M.	......	9 4	......	0·823
	10.	4 30	,,	10 9	......	0·651	
	10.	12 30	P.M.	......	8 0	......	0·709
	10.	6 45	,,	12 2	......	0·646	
	11.	2 0	A.M.	......	9 6		
	11.	6 40	,,	11 0	......		

A. *Diurnal Tide.*

The general expression for the Diurnal Tide is

$$D=M\sin 2\bar{\mu}\cos(m-i_m)+S\sin 2\bar{\sigma}\cos(s-i_s), \quad \ldots \ldots \quad (1)$$

which at the Equinoxes reduces simply to the Lunar Tide, viz.

$$D=M\sin 2\bar{\mu}\cos(m-i_m). \quad \ldots \ldots \quad (2)$$

If the Tides be plotted carefully to scale, it appears that the Diurnal Tides in height vanish together at High Water and Low Water, when $\mu=0$, or nearly so.

The mean interval from the time of the Moon's declination vanishing to the disappearance of the Diurnal Inequality is about 36 hours, which may be regarded as the age of the Lunar Diurnal Tide. It is evident from equation (2) that if h and l represent the range of Tide at High Water and Low Water respectively, since the phase changes by 90° from High Water to Low Water, we have the following equations to determine the unknown constants i_m and M:—

$$\cot(m-i_m)=\pm\frac{h}{l}, \quad \ldots \ldots \quad (3)$$

$$2M\sin 2(\overline{\text{max. value of }\mu})=\sqrt{h^2+l^2}. \quad \ldots \ldots \quad (4)$$

The mean maximum values of h and l were found to be

$$h=0\cdot849 \text{ foot},$$
$$l=0\cdot761 \text{ foot};$$

hence we find

$$\cot(m-i_m)=\pm\frac{849}{761},$$

$$m-i_m=41^\circ 52' \text{ or } -138^\circ 8'$$
$$=2^h 53^m \text{ or } -9^h 7^m. \quad \ldots \ldots \quad (5)$$

The mean values of m at High Water and Low Water, as appears from the following Table, are:—

		h m
High Water	$m=$	−0 27
Low Water	$m=$	6 1

or, reducing both to High-Water Standard,

$$m = -0^h\,27^m$$
$$\quad -0^h\,11^m$$
$$\text{Mean} \ldots -0^h\,19^m$$

Hence, by equation (5),

$$-0^h\,19^m - i_m = 2^h\,53^m,$$
$$i_m = -3^h\,12^m,$$

or

$$-0^h\,19^m - i_m = -9^h\,7^m,$$
$$i_m = +8^h\,48^m.$$

An examination of the signs of the Diurnal Tide shows that we must select the value

$$i_m = +8^h\,48^m \quad \ldots \ldots \quad 5\ (bis)$$

From equation (4) we find

$$M = \frac{\sqrt{(0{\cdot}849)^2 + (0{\cdot}761)^2}}{2 \sin 49^\circ}$$
$$= 0{\cdot}76 \text{ foot} = 9{\cdot}06 \text{ inches} \quad \ldots \ldots \quad (6)$$

If we plot the Lunitidal Intervals at High Water and Low Water to scale, from the following Table we obtain the Diurnal Inequality in time. It produces a maximum acceleration or retardation in the time of Tide, amounting to 39 minutes.

The following Table gives the Lunitidal Intervals at High Water and Low Water.

TABLE II.—Refuge Cove. Lunitidal Intervals.

1853.	h m	High Water. h m	Low Water. h m	1853.	h m	High Water. h m	Low Water. h m
Sept. 17.	6 30 A.M.	......	6 35	Sept. 21.	8 30 A.M.	......	6 19
17.	12 30 P.M.	+0 11		21.	2 25 P.M.	−0 48	
17.	6 25 „	......	5 54	21.	8 40 „	......	6 33
18.	1 0 A.M.	+0 20		22.	2 54 A.M.	−0 39	
18.	7 30 „	......	5 10	22.	8 45 „	......	6 48
18.	12 50 P.M.	−0 14		22.	3 30 P.M.	−0 27	
18.	7 0 „	......	6 4	22.	9 45 „	......	6 12
19.	1 30 A.M.	+0 7		23.	3 30 A.M.	−0 49	
19.	7 40 „	......	5 43	23.	9 50 „	......	6 29
19.	1 45 P.M.	−0 2		23.	3 55 P.M.	−0 48	
19.	7 55 „	......	5 52	23.	10 30 „	......	6 13
20.	2 0 A.M.	−0 6		24.	4 0 A.M.	−1 6	
20.	8 20 „	......	5 46	24.	10 40 „	......	6 26
20.	2 10 P.M.	−0 20		24.	5 0 P.M.	−0 30	
20.	8 0 „	......	6 30	24.	11 10 „	.....	6 20
21.	2 5 A.M.	−0 44		25.	5 10 A.M.	−0 45	

TABLE II. (continued).

	High Water.	Low Water.		High Water.	Low Water.
1853. h m	h m	h m	1853. h m	h m	h m
Sept. 25. 10 20 A.M.	......	7 35	Oct. 3. 6 45 P.M.	......	5 51
25. 5 30 P.M.	−0 49		3. Midnight	−1 0	
26. 12 30 A.M.	......	5 49	4. 7 15 A.M.	......	5 45
26. 5 45 „	−1 1		4. Noon	−1 27	
26. 12 30 P.M.	......	6 16	4. 8 0 P.M.	......	5 27
26. 6 30 „	−0 40		5. 1 0 A.M.	−0 51	
27. 1 15 A.M.	......	5 55	5. 8 30 „	......	5 21
27. 7 0 „	−0 37		5. 1 28 P.M.	−0 53	
27. 1 30 P.M.	......	6 7	5. 8 20 „	......	6 1
27. 7 30 „	−0 31		6. 1 40 A.M.	−1 5	
28. 3 0 A.M.	......	5 1	6. 8 30 „	......	6 15
28. 8 0 „	−0 29		6. 1 0 P.M.	−2 17	
28. 3 30 P.M.	......	4 59	6. 9 30 „	......	5 47
28. 10 5 „	+1 12		7. 4 0 A.M.	+0 19	
29. 4 0 A.M.	......	4 53	7. 9 0 „	......	6 41
29. 10 25 „	+1 6		7. 3 0 P.M.	−1 16	
29. 4 0 P.M.	......	5 19	7. 9 45 „	......	6 31
29. 10 40 „	+0 57		8. 4 0 A.M.	−0 40	
30. 5 0 A.M.	......	4 43	8. 10 0 „	......	6 40
30. 10 45 „	+0 36		8. 5 0 P.M.	−0 17	
30. 4 45 P.M.	......	5 24	8. 10 50 „	......	6 27
30. 11 45 „	+1 12		9. 4 15 A.M.	−1 26	
Oct. 1. 6 0 A.M.	......	4 33	9. 10 50 „	......	6 51
1. 11 0 „	+0 2		9. 4 50 P.M.	−1 28	
1. 5 15 P.M.	......	6 17	10. 12 10 A.M.	......	6 8
1. 11 45 „	+0 23		10. 4 30 „	−2 12	
2. 5 0 A.M.	......	6 22	10. 12 30 P.M.	......	6 12
2. 12 30 P.M.	+0 18		10. 6 45 „	−0 31	
2. 6 15 „	......	5 57	11. 2 0 A.M.	......	5 16
3. 12 20 A.M.	−0 16		11. 6 40 „	−1 0	
3. 6 0 „	......	6 36			
3. 1 0 P.M.	+0 24		Mean..................	−0 26·7	6 1·1

If we compare the mean Lunitidal Intervals here found with the corresponding intervals at Northumberland Sound, we find:—

Mean Lunitidal Interval.

	High Water.	Low Water.
	h m	h m
Refuge Cove	−0 26·7	6 1·1
Northumberland Sound . .	+0 7·05	6 35·35
Difference	33·75	34·25

These differences represent the time of the Atlantic Tide-wave passing from Refuge Cove to Northumberland Sound (uncorrected for longitude); and their agreement is a proof of the accuracy of the observations at both places.

B. *Semidiurnal Tide* (Heights).

When Table I. is plotted to scale, it is easy to correct the tide for the Diurnal Inequality, or to do so by the Diurnal Tide at High Water and Low Water given in that Table. When this correction is made we find the following Spring and Neap Ranges:—

		ft.	in.
Springs.—19th September,	1.30 A.M.	5	8
5th October,	1.28 P.M.	5	$8\frac{1}{2}$
Neaps.— 27th September,	7.0 A.M.	1	4

Using the formula for the Semidiurnal Tide,

$$T = M' \cos 2(m - i_m) + S' \cos 2(s - i_s), \qquad (7)$$

$$T = A \cos 2(m - B),$$

we find at Springs

$$(M' + S') = 34{\cdot}1 \text{ inches},$$

and at Neaps

$$(M' - S') = 8 \text{ inches};$$

from which we obtain

$$2M' = 42{\cdot}1 \text{ inches},$$

$$2S' = 26{\cdot}1 \quad ,,$$

$$\frac{S'}{M'} = 0{\cdot}621.$$

C. *Semidiurnal Tide* (Intervals).

When Table II. is plotted to scale, and the Tide corrected for the Diurnal Inequality, we obtain the following results, making use of the formulæ given in discussing the Tide at Northumberland Sound:—

Maximum Value of 2 B.

Range of Lunitidal Interval at High Water.

		h	m
29th September,	10.25 A.M.	+1	6
9th October,	4.50 P.M.	−1	24
	2 B=	2	30

Range of Lunitidal Interval at Low Water.

		h	m
25th September,	10.20 A.M.	+6	48
28th September,	3.30 P.M.	+4	59
	2 B=	1	49

The approximate value of i_m, taken from the mean of the observations, is,

	h	m
At High Water	−0	27
Low Water	−0	11

Hence we have

$$2\,(B-i_m).$$

	h m	
High Water	3 24	49° 30′
Low Water	2 3	30 0

Hence we obtain

$$\frac{S'}{M'}=\sin 2\,(B-i_m)=0{\cdot}76 \text{ High Water}$$

$$=0{\cdot}50 \text{ Low Water.}$$

$$\text{Mean} \quad . \quad . \quad . \quad . \quad \overline{0{\cdot}63}$$

Collecting the several constants, we obtain:—

Diurnal Tide.

$$i_m=+8^h\,48^m,$$

$$M'=\quad 9{\cdot}06 \text{ inches.}$$

Semidiurnal Tide.

$$\frac{S'}{M'}=0{\cdot}62 \text{ (Heights),}$$

$$\frac{S'}{M'}=0{\cdot}63 \text{ (Intervals).}$$

	High Water.	Low Water.
Mean Lunitidal Interval=	$-0^h\,26^m{\cdot}7$	$6^h\,1^m{\cdot}1.$